MENIIT
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JEE Advanced : Paper-1 (2014)

IMPORTANT INSTRUCTIONS

A. General:

1. This booklet is your Question Paper. Do not break the seal of this booklet before being instructed to do so by the invigilators.
2. The question paper CODE is printed on the left hand top corner of this sheet and on the back cover page of this booklet.
3. Blank space and blank pages are provided in the question paper for your rough work. No additional sheets will be provided for rough work.
4. Blank papers, clipboards, log tables, slide rules, calculators, cameras, cellular phones, pagers and electronic gadget of any kind are NOT allowed inside the examination hall.
5. Write your name and roll number in the space provided on the back cover of this booklet.
6. Answers to the questions and personal details are to be filled on an Optical Response Sheet, which is provided separately.

   The ORS is a doublet of two sheets – upper and lower, having identical layout. The upper sheet is a machine-gradable

   Objective Response Sheet (ORS) which will be collected by the invigilator at the end of the examination. The upper sheet is designed in such a way that darkening the bubble with a ball point pen will leave an identical impression at the corresponding place on the lower sheet. You will be allowed to take away the lower sheet at the end of the examination (see Figure-1 on the back cover page for the correct way of darkening the bubbles for valid answers).
7. Use a black ball point pen only to darken the bubbles on the upper original sheet. Apply sufficient pressure so that the impression is created on the lower sheet. **See Figure -1** on the back cover page for appropriate way of darkening the bubbles for valid answers.
8. DO NOT TAMPER WITH / MUTILATE THE ORS SO THIS BOOKLET.
9. On breaking the seal of the booklet check that it contains 28 pages and all the 60 questions and corresponding answer choices are legible. Read carefully the instruction printed at the beginning of each section.

B. Filling the right part of the ORS

10. The ORS also has a CODE printed on its left and right parts.
11. Verify that the CODE printed on the ORS (on both the left and right parts) is the same as that on the this booklet and put your signature in the Box designated as R4.
12. **IF THE CODES DO NOT MATCH, ASK FOR A CHANGE OF THE BOOKLET / ORS AS APPLICABLE.**
13. Write your Name, Roll No. and the name of centre and sign with pen in the boxes provided on the upper sheet of ORS. Do not write any of this anywhere else. Darken the appropriate bubble UNDER each digit of your Roll No. in such way that the impression is created on the bottom sheet. (see example in Figure 2 on the back cover)

C. Question Paper Format

The question paper consists of three parts (Physics, Chemistry and Mathematics). Each part consists of two sections.

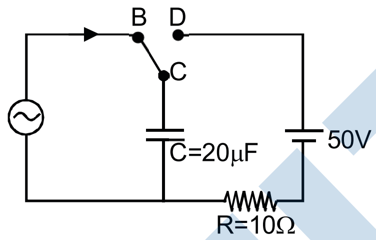
14. **Section 1** contains **10 multiple** choice questions. Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE THAN ONE** are correct.
15. **Section 2** contains **10 questions.** The answer to each of the questions is a single-digit integer, ranging from 0 to 9 (both inclusive)

PART A : PHYSICS

(One or More Than One Options Correct Type)

This section contains **10 multiple choice questions**. Each question has four choice (A), (B), (C) and (D) out of which **ONL or More THAN ONE** are correct.

1. At time t = 0, terminal A in the circuit shown in the figure is connected to B by a key and an alternating current $I(t) = I_0 \cos(\omega t)$, with I_0 = 1A and ω = 500 rad s^{-1} starts flowing in it with the initial direction shown in the figure. At $t = \frac{7\pi}{6\omega}$, they key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If C = 20 μF, R = 10Ω and the battery is ideal with emf of 50V, identify the correct statement(s).



(A) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is 1×10^{-3} C.

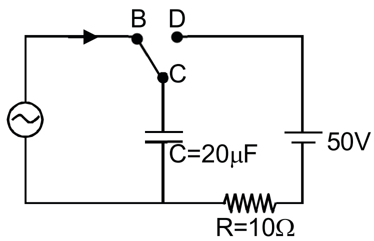
(B) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise.

(C) Immediately after A is connected to D, the current in R is 10 A.

(D) $Q = 2 \times 10^{-3}$ C.

Ans. **[CD]**

Sol. $I = I_0 \cos\omega t = \cos\left(\frac{7\pi}{6\omega}\right) = -\frac{\sqrt{3}}{2}$

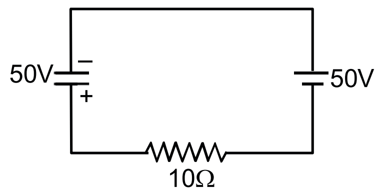


So direction of current is anticlockwise.

$Q = \int I dt = \int_0^{\frac{7\pi}{6\omega}} \cos\omega t \, dt = 10^{-3}$ C (Lower plate positive)

Final charge on capacitor = $20 \times 10^{-6} \times 50 = 10^{-3}$ C (Upper plate positive)

So, max. charge flown = $\frac{I_0}{\omega}$ = 2×10^{-3} C



$V = \frac{10^{-3}}{20 \times 10^{-6}} = \frac{10^3}{20} = \frac{100 \times 10}{20} = 50$ V

So, current = $\frac{100}{10}$ = 10 A

2. A light source, which emits two wavelength λ_1 = 400 nm and λ_2 = 600 nm, is used in a Young's double slit experiment. If recorded fringe widths for λ_1 and λ_2 are β_1 and β_2 and the number of fringes for them within a distance y on one side of the centre maximum are m_1 and m_2, respectively, then :

(A) $\beta_2 > \beta_1$

(B) $m_1 > m_2$

(C) From the centre maximum, 3rd maximum of λ_2 overlaps with 5th minimum of λ_1

(D) The angular separation of fringes for λ_1 is greater than λ_2

Ans. **[ABC]**

Sol. $\beta_1 = \frac{\lambda_1 D}{d}$

$\beta_2 = \frac{\lambda_2 D}{d}$

$y = \frac{m_1\lambda_1 D}{d}$

$m_1 > m_2$

3^{rd} max $\frac{3\lambda_2 D}{d} = \frac{3 \times 600D}{d}$

5^{th} min $\frac{4.5\lambda_1 D}{d} = 4.5 \times \frac{400D}{d}$

$\theta = \frac{\lambda}{d}$

3. One end of a taut string of length 3m along the x-axis is fixed at x = 0. The speed of the waves in the string is 100 ms^{-1}. The other end of the string is vibrating in the y direction so that stationary waves are set up in the string. The possible wave from(s) of these stationary waves is(are)

(A) $y(t) = A \sin\frac{\pi x}{6}\cos\frac{50\pi t}{3}$

(B) $y(t) = A \sin\frac{\pi x}{3}\cos\frac{100\pi t}{3}$

(C) $y(t) = A \sin\frac{5\pi x}{6}\cos\frac{250\pi t}{3}$

(D) $y(t) = A \sin\frac{5\pi x}{2}\cos 250\pi t$

Ans. **[ACD]**

Sol. $(2p - 1)\frac{\lambda}{4} = 3$ $\therefore$ $\lambda = \frac{12}{2p-1}$

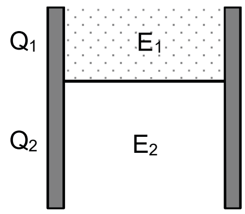
$c = f\lambda \Rightarrow 100 = f\,\frac{(12)}{2p-1}$

$\therefore f = (2p - 1)\frac{100}{12}$

$k = \frac{2\pi}{\lambda} = \frac{\pi}{6}(2p - 1)$

$\omega = 2\pi f = \frac{50\pi(2p-1)}{3}$

4. A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. The electric field in the dielectric is E_1 and that in the other portion is E_2. Choose the correct option/options, ignoring edge effects.



(A) $\frac{E_1}{E_2} = 1$ (B) $\frac{E_1}{E_2} = \frac{1}{K}$ (C) $\frac{Q_1}{Q_2} = \frac{3}{K}$ (D) $\frac{C}{C_1} = \frac{2+K}{K}$

Ans. **[AD]**

Sol. $\frac{k \in_0 \frac{A}{3}}{d} = C_1$

$\frac{k \in_0 \frac{A}{3}}{d} + \frac{2 \in_0 \frac{A}{3}}{d} = C$

$Q_1 = C_1V = \frac{k \in_0 A}{3d} V$

$Q_2 = (C - C_1)\, V = \frac{2 \in_0 A}{3d} V$

$\frac{E_1}{E_2} = \frac{\frac{V}{d}}{\frac{V}{d}} = 1$

5. Let $E_1(r)$, $E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance r from a point charge Q, an infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, then

(A) $Q = 4\sigma\pi r_0^2$ (B) $r_0 = \frac{\lambda}{2\pi\sigma}$

(C) $E_1(r_0/2) = 2E_2\,(r_0/2)$ (D) $E_2\,(r_0/2) = 4E_3(r_0/2)$

Ans. **[C]**

Sol. (C) $\frac{Q}{4\pi\varepsilon_0 r_0^2} = \frac{\lambda}{2\pi\varepsilon_0 r_0} = \frac{\sigma}{2\varepsilon_0}$

$E_1\left(\frac{r_0}{2}\right) = \frac{Q}{4\pi\varepsilon_0 (r_0/2)^2}$

$$2E_2\left(\frac{r_0}{2}\right)=\frac{2\lambda}{2\pi\varepsilon_0(r_0/2)}$$

(D) $E_2\left(\frac{r_0}{2}\right)=\frac{\lambda}{2\pi\varepsilon_0(r_0/2)}$

$$4E_3\left(\frac{r_0}{2}\right)=\frac{4\sigma}{2\varepsilon_0}$$

6. A student is performing an experiment using a resonance column and a tuning fork of frequency 244 s^{-1}. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is (0.350 ± 0.005)m, the gas in the tube is

(**Useful information:** $\sqrt{167RT}$ = 640 $J^{1/2}$ $mole^{-1/2}$; $\sqrt{140RT}$ = 590 $J^{1/2}$ $mole^{-1/2}$. The molar masses M in grams are given in the options. Take the values of $\sqrt{\frac{10}{M}}$ for each gas as given there.)

(A) Neon $\left(M=20,\sqrt{\frac{10}{20}}=\frac{7}{10}\right)$

(B) Nitrogen $\left(M=28,\sqrt{\frac{10}{28}}=\frac{3}{5}\right)$

(C) Oxygen $\left(M=32,\sqrt{\frac{10}{32}}=\frac{9}{16}\right)$

(D) Argon $\left(M=36,\sqrt{\frac{10}{36}}=\frac{17}{32}\right)$

Ans. **[D]**

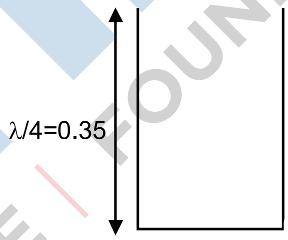
Sol. λ = 1.4

v = fλ = 244 × 1.4 = 341.6

error $=\frac{5}{350}\simeq 1.3\%$

v = 341.6 ± 4.88

336.72 < v < 346.48



$$v_{mono}=\sqrt{\frac{\gamma RT}{M}}=\sqrt{\frac{1.67RT}{20\times10^{-3}}}$$

$$v_{Ne}=\sqrt{167RT}\times\sqrt{\frac{10}{20}}=640\times\frac{7}{10}=448$$

7. Heater of an electric kettle is made of a wire of length L and diameter d. It takes 4 minutes to raise the temperature of 0.5 kg water by 40K. This heater is replaced by a new heater having two wires of the same material, each of length L and diameter 2d. The way these wires are connected is given in the options. How much time in minutes will it take to raise the temperature of the same amount of water by 40K?

(A) 4 if wires are in parallel

(B) 2 if wires are in series

(C) 1 if wires are in series

(D) 0.5 if wires are in parallel

Ans. **[BD]**

Sol. $\frac{dQ}{dt}=ms\frac{dT}{dt}=\frac{V^2}{R}=P_0$

$$R_1=\frac{R}{4}$$

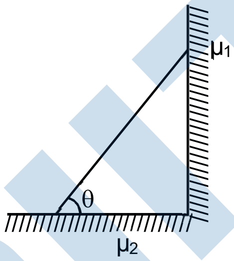
$\Rightarrow$ if series $R_{eq} = \frac{R}{2}$

if parallel $R_{eq} = \frac{R}{8}$

$\Rightarrow$ $P_{series} = \frac{2V^2}{R} = 2P_0$

$P_{parallel} = \frac{8V^2}{R} = 8P_0$

8. In the figure, a ladder of mass m is shown leaning against a wall. It is in static equilibrium making an angle θ with the horizontal floor. The coefficient of friction between the wall and the ladder is μ_1 and that between the floor and the ladder is μ_2. The normal reaction of the wall on the ladder is N_1 and that of the floor is N_2. If the ladder is about to slip, then



(A) $\mu_1 = 0$ $\mu_2 \neq 0$ and $N_2 \tan\theta = \frac{mg}{2}$

(B) $\mu_1 \neq 0$ $\mu_2 = 0$ and $N_1 \tan\theta = \frac{mg}{2}$

(C) $\mu_1 \neq 0$ $\mu_2 \neq 0$ and $N_2 = \frac{mg}{1+\mu_1\mu_2}$

(D) $\mu_1 = 0$ $\mu_2 \neq 0$ and $N_1 \tan\theta = \frac{mg}{2}$

Ans. **[CD]**

Sol. $\mu_2 N_2 = N_1$

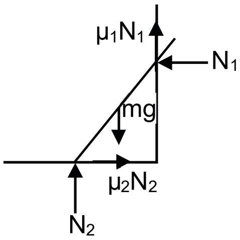
$\mu_1 N_1 + N_2 = mg$

$mg\frac{L}{2}\cos\theta = N_1 L\sin\theta + \mu_1 N_1 \cos\theta$

if $\mu_1 = 0$, $N_2 = mg$

$N_1 = \frac{mg}{2}\cot\theta$

$\Rightarrow$ $N_1 \tan\theta = \frac{mg}{2}$



if $\mu_2 = 0$, $N_1 = 0$

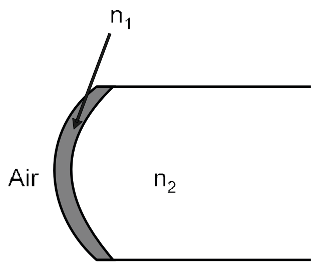
$N_2 = mg$

$\Rightarrow$ not possible

$\mu_1 \neq 0$, $\mu_2 \neq 0$

$N_2 = \frac{mg}{1+\mu_1\mu_2}$

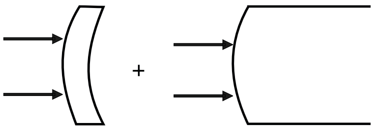
9. A transparent thin film of uniform thickness and refractive index n_1 = 1.4 is coated on the convex spherical surface of radius R at one end of a long solid glass cylinder of refractive index n_2 = 1.5, as shown in the figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at distance f_1 from the film, while rays of light traversing from glass to air get focused at distance f_2 from the film. Then



(A) $|f_1| = 3R$ (B) $|f_1| = 2.8R$ (C) $|f_2| = 2R$ (D) $|f_2| = 1.4R$

Ans. **[AC]**

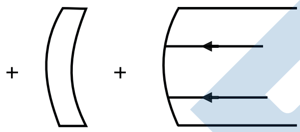
Sol.



$$\frac{1}{f_1} = (n_1 - 1)\left(\frac{1}{R} - \frac{1}{R}\right) = 0$$

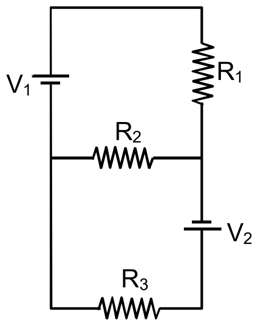
$$\frac{n_1}{v} - \frac{1}{\infty} = \frac{n_2 - 1}{R} = \frac{1}{2R}$$

$v = 3R \Rightarrow f_1 = 3R$



$$\frac{1}{v} - \frac{n_2}{\infty} = \frac{1 - n_2}{-R}$$

10. Two ideal batteries of emf V_1 and V_2 and three resistances R_1, R_2 and R_3 are connected as shown in the figure. The current in resistance R_2 would be zero if

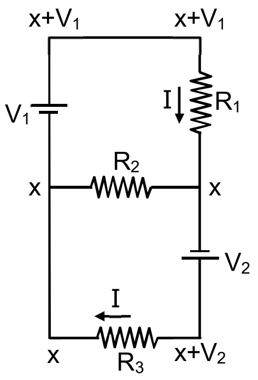


(A) $V_1 = V_2$ and $R_1 = R_2 = R_3$ (B) $V_1 = V_2$ and $R_1 = 2R_2 = R_3$

(C) $V_1 = 2V_2$ and $2R_1 = 2R_2 = R_3$ (D) $2V_1 = V_2$ and $2R_1 = R_2 = R_3$

Ans. **[ABD]**

Sol.

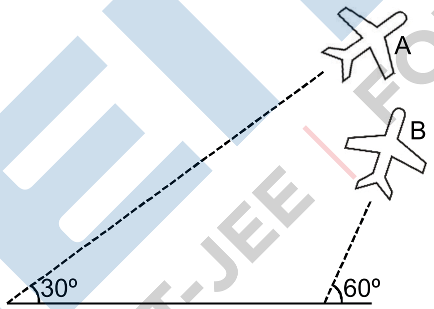


$$\frac{V_1}{R_1} = \frac{V_2}{R_3}$$

Section – II : (One Integer value correct Type)

This section contains **10 questions**. Each question, when worked out will result in one integer from 0 to 9 (both inclusive).

11. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in figure. The speed of A is $100\sqrt{3}\, ms^{-1}$. At time t = 0s, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at t = t_0, A just escapes being hit by B, t_0 in seconds is :



Ans. **[5]**

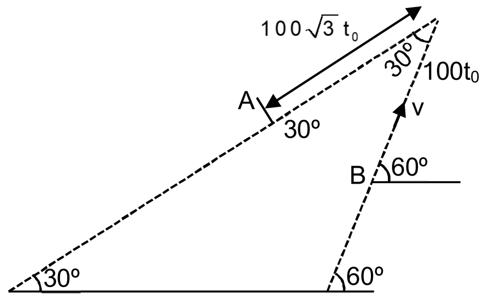
Sol. $V_{BA} = \vec{V}_B - \vec{V}_A$

$\Rightarrow$ v cos 30° $= 100\sqrt{3}$

$\Rightarrow$ v = 200 m/s

V_{BA} = v sin 30° = 100 m/s

t = S_{BA}/V_{BA} = 5 sec.



12. During Searle's experiment, zero of the Vernier scale lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale. The 20^{th} division of the Vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the Vernier scale still lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale but now the 45^{th} division of Vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2 m and its cross-sectional area is 8×10^{-7} m^2. The least count of the Vernier scale is 1.0×10^{-5} m. The maximum percentage error in the Young's modulus of the wire is

Ans. [4]

Sol. $Y = \frac{FL}{A\Delta L}$

$\frac{dY}{Y} = \frac{d(\Delta L)}{\Delta L}$

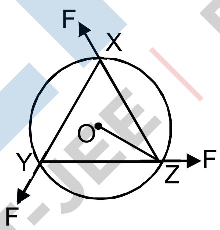
$x_1 = 3.20 \times 10^{-2} + 20 \times 10^{-5} = 3.220$

$x_2 = 3.20 \times 10^{-2} + 4.5 \times 10^{-5} = 3.245 \times 10^{-2}$ m

$\therefore \Delta L = 0.025 \times 10^{-2}$ m

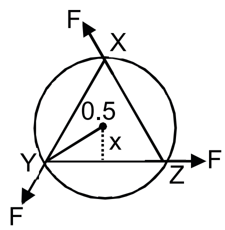
$\therefore \frac{dY}{Y} = \frac{10^{-5}}{0.025 \times 10^{-2}} \times 100 = 4$

13. A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude F = 0.5 N are applied simultaneously along the three sides of an equilateral triangle XYZ with its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc in rad s^{-1} is



Ans. [2]

Sol. $x = 0.5 \sin 30° = \frac{1}{4}$ m



$\tau = F \times \frac{1}{4} + F \times \frac{1}{4} + F \times \frac{1}{4} = I\alpha$

$3 \times \frac{0.5}{4} = \frac{1}{2} \times 1.5 \times (0.5)^2 \alpha$

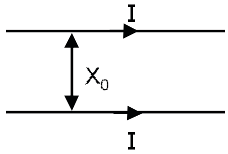
$\alpha = 2$ rad/s^2

$\omega = \alpha t = 2$ rad/s

14. Two parallel wires in the plane of the paper are distance X_0 apart. A point charge is moving with speed u between the wires in the same plane at a distance X_1 from one of the wires. When the wires carry current of magnitude I in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast, if the currents I in the two wires have directions opposite to each other, the radius of curvature of the path is R_2. If $\frac{x_0}{x_1} = 3$, the value of $\frac{R_1}{R_2}$ is

Ans. [3]

Sol. $R_1 = \frac{mv}{qB} = \frac{mu}{qB}$



$$B = \frac{\mu_0 i}{2\pi x_1} - \frac{\mu_0 i}{2\pi(x_0 - x_1)} = \frac{\mu_0 i}{2\pi x_1} \times \left(1 - \frac{1}{2}\right)$$

$$B' = \frac{\mu_0 i}{2\pi x_1} + \frac{\mu_0 i}{2\pi(x_0 - x_1)} = \frac{\mu_0 i}{2\pi x_1}\left(1 + \frac{1}{2}\right)$$

$$\Rightarrow \frac{R_1}{R_2} = \frac{B'}{B} = 3$$

15. To find the distance d over which a signal can be seen clearly in foggy conditions, a railways engineer uses dimensional analysis and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer finds that d is proportional to $S^{1/n}$. The value of n is

Ans. [3]

Sol. $d = k\,\rho^x s^y f^z$

$L = [ML^{-3}]^x [MT^{-3}]^y [T^{-1}]^z$

$x + y = 0$

$-3y - z = 0$

$1 = -3x \Rightarrow x = \frac{-1}{3}, y = \frac{1}{3}$

$\Rightarrow x = 3$

16. A galvanometer gives full scale deflection with 0.006 A current. By connecting it to a 4990 Ω resistance, it can be converted into a voltmeter of range 0 - 30 V. If connected to a $\frac{2n}{249}\Omega$ resistance, it becomes an ammeter of range 0 - 1.5 A. The value of n is

Ans. [5]

Sol. $V = i_g (R + G)$

$30 = 0.006\,(4990 + G)$

$G = 10$

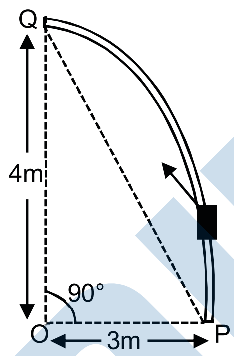
$$\frac{i(RG)}{R+G} = i_g G$$

$$i = \frac{(R+G)i_g}{R}$$

$$1.5 = \frac{\left(10+\frac{2n}{249}\right)0.006}{\frac{2n}{249}}$$

n = 5

17. Consider an elliptically shaped rail PQ in the vertical plane with OP = 3 m and OQ = 4m. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ (see the figure given). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is (n × 10) Joules. The value of n is (take acceleration due to gravity = 10 ms^{-2})



Ans. 5

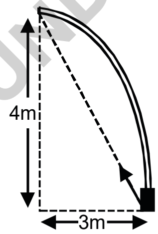
Sol. F is constant

$\Rightarrow W_F = F \times 5 = 90$ J

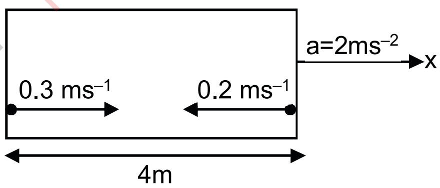
$w_{gr} = -mgh = -1 \times 10 \times 4 = -40$ J

90 – 40 = K = 50

n = 5



18. A rocket is moving in a gravity free space with a constant acceleration of 2 ms^{-2} along + x direction (see figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in +x direction with a speed of 0.3 ms^{-1} relative to the rocket. At the same time, another ball is thrown in –x direction with a speed of 0.2 ms^{-1} from its right end relative to the rocket. The time in seconds when the two balls hit each other is

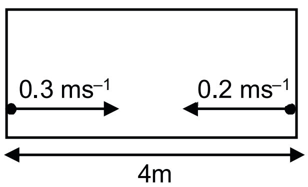


Ans. **[8]** **[JEE Answer 2 or 8]**

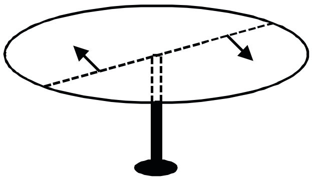
Sol. $V_{rel} = 0.5$ m/s

$a_{rel} = 0$

$$t = \frac{S_{rel}}{V_{rel}} = \frac{4}{0.5} = 8\,\text{sec}$$



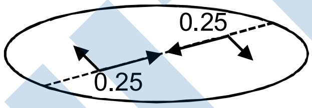
19. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0.25 m from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 ms^{-1} with respect to the ground. The rotational speed of the platform in rad s^{-1} after the balls leave the platform is :



Ans. **[4]**

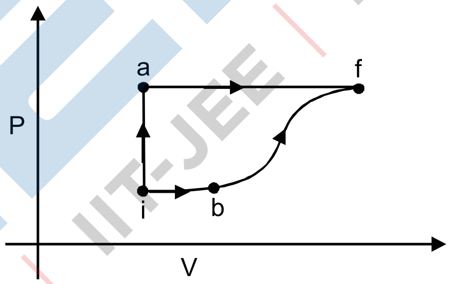
Sol. L = const.

$0 = 0.05 \times 9 \times 0.25 \times 2 - \frac{1}{2} \times 0.45\,(0.5)^2\omega$



ω = 4 rad/s

20. A thermodynamic system is taken from an initial state i with internal energy U_i = 100 J to the final state f along two different paths iaf and ibf, as schematically shown in the figure. The work done by the system along the paths af, ib and bf are W_{af} = 200 J, W_{ib} = 50 J and W_{bf} = 100 J respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{iaf}, Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state b is U_b = 200 J and Q_{iaf} = 500 J, the ratio Q_{bf} / Q_{ib} is :



Ans. **[2]**

Sol. $500 = U_f - 100 + 200 + 0$

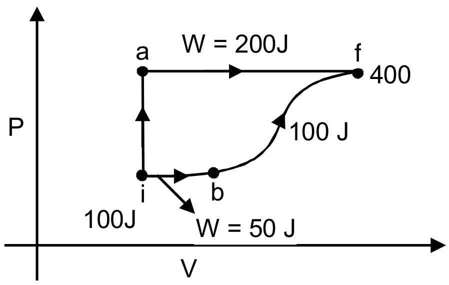
U_f = 400 J

$Q_{ib} = W + \Delta U = 50 + (200 - 100) = 150$ J

$Q_{bf} = W_{bf} + 400 - 200$

= 100 + 200 = 300

$\frac{Q_{bf}}{Q_{ib}} = 2$



PART B : CHEMISTRY

SECTION – 1

(One or More Than one option correct Type)

This section contains **10 multiple choice type questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE** or MORE THAN ONE are correct.

21. The correct combination of names for isomeric alcohols with molecular formula $C_4H_{10}O$ is/are

(A) tert-butanol and 2-methylpropan-2-ol
(B) tert-butanol and 1,1-dimethylethan-1-ol
(C) n-butanol and butan-1-ol
(D) isobutyl alcohol and 2-methylpropan-1-ol

Ans. **[ABCD]**

Sol. $C_4H_{10}O \rightarrow$ du = 0

(A) $CH_3-C(CH_3)(OH)-CH_3$ tert-butyl alcohol; $CH_3-C(CH_3)(OH)-CH_3$ 2-methylpropan-2-ol

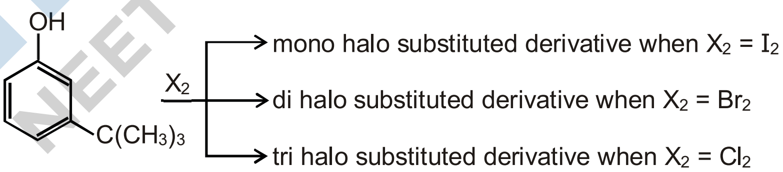
(B) $CH_3-C(CH_3)(OH)-CH_3$ tert-butanol; $CH_3-\boxed{C-CH_3}(CH_3)(OH)$ 1,1, dimethylethan-1-ol

(C) $CH_3–CH_2–CH_2–CH_2–OH$ n-Butanol; $CH_3–CH_2–CH_2–CH_2–OH$ Butan–1–ol

(D) $CH_3-CH(CH_3)-CH_2-OH$ isobutyl alcohol; $\overset{3}{CH_3}-\overset{2}{CH}(CH_3)-\overset{1}{CH_2}-OH$ 2-methylpropan-1-ol

So all are isomeric alcohol with molecular formula $C_4H_{10}O$

22. The reactivity of compound Z with different halogens under appropriate conditions is given below :

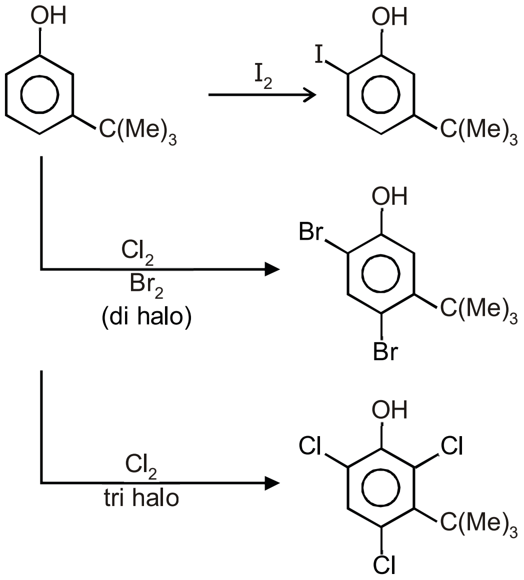


The observed pattern of electrophilic substitution can be explained by

(A) the steric effect of the halogen
(B) the steric effect of the tert-butyl group
(C) the electronic effect of the phenolic group
(D) the electronic effect of the tert-butyl group

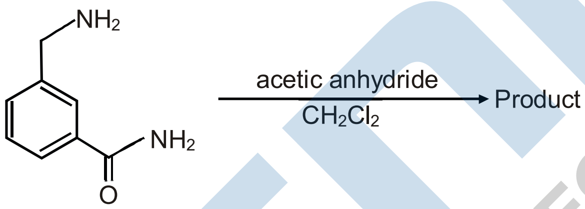
Ans. **[ABC]**

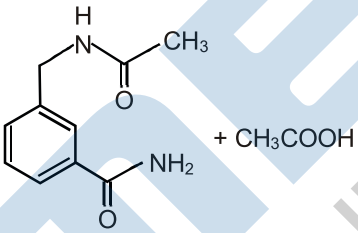
Sol.

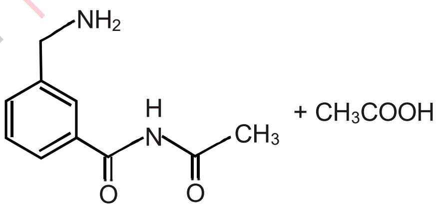


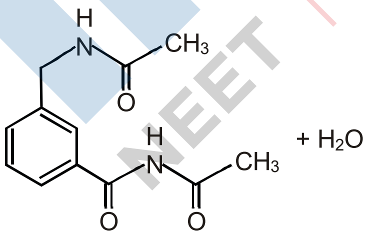
OH is strong activating +M group, so substitution of electrophile take place o/p to OH group and in substitution stearic hindrance of tert-butyl and Halogen also plays a role

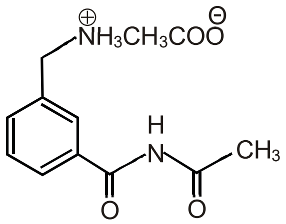
23. In the reaction shown below, the major product(s) formed is/are



(A) 

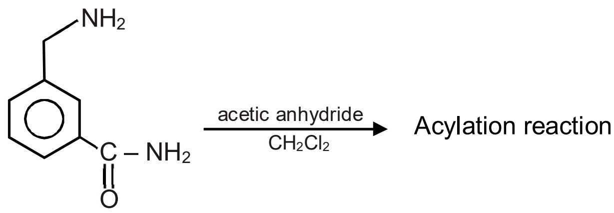
(B) 

(C) 

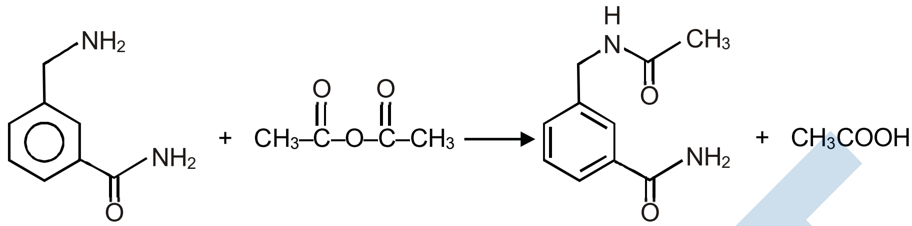
(D) 

Ans. **[A]**

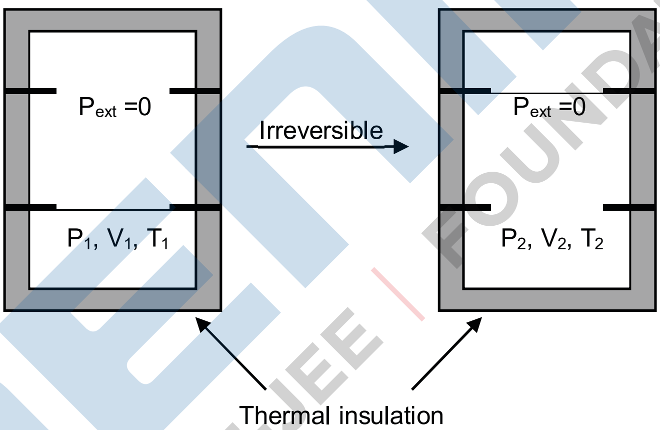
Sol.



Acylation takes place on strong nucleophilic part on aliphatic amine. Amide is weak nucleophile so no acylation fissible.



24. An ideal gas in a thermally insulated vessel at internal pressure = P_1, volume = V_1 and absolute temperature = T_1 expands irreversibly against zero external pressure, as shown in the diagram. The final internal pressure, volume and absolute temperature of the gas are P_2 , V_2 and T_2 , respectively. For this expansion,



(A) $q = 0$ (B) $T_2 = T_1$

(C) $P_2V_2 = P_1V_1$ (D) $P_2V_2^{\gamma} = P_1V_1^{\gamma}$

Ans. **[ABC]**

Sol. $\because P_{ext} = 0$

$\therefore w = 0 \because$ thermally insulated $\therefore q = 0$

$\therefore \Delta U = 0$, hence $T_2 = T_1$

$\therefore P_1V_1 = P_2V_2$

$\therefore$ [A], [B], [C]

25. Hydrogen bonding plays a central role in the following phenomena :

(A) Ice floats in water.

(B) Higher Lewis basicity of primary amines than tertiary amines in aqueous solutions

(C) Formic acid is more acidic than acetic acid.

(D) Dimerisation of acetic acid in benzene.

Ans. **[ABD]**

Sol. [A] Due to the intermolecular hydrogen bonding it forms cage like structure so effective volume increases and Ice floats in water.

[B] Higher Lewis basicity of primary amines than tertiary amines in aqueous solutions due to the more hydration in primary amine

[C] Formic acid is more acidic than acetic acid due to +I effect of CH_3– group in CH_3COOH

[D] It exist as a dimeric form due to the formation of intermolecular hydrogen bond.

26. In a galvanic cell, the salt bridge

(A) does not participate chemically in the cell reaction.

(B) stops the diffusion of ions from one electrode to another

(C) is necessary for the occurrence of the cell reaction.

(D) ensures mixing of the two electrolytic solution.

Ans. **[AB]**

Sol. [A] $\because$ Salt Bridge consists of non interfering ions with those involved in reaction.

[B] restricts diffusion of ions

[C] reaction can occur without salt bridge using porous membrane

27. Upon heating with Cu_2S, the reagent(s) that give copper metal is/are :

(A) $CuFeS_2$ (B) CuO (C) Cu_2O (D) $CuSO_4$

Ans. **[C]**

Sol. $Cu_2S + 2Cu_2O \xrightarrow{\Delta} 6Cu + SO_2\uparrow$

28. The correct statement(s) for orthoboric acid is/are

(A) It behaves as a weak acid in water due to self ionization.

(B) Acidity of its aqueous solution increases upon addition of ethylene glycol

(C) It has a three dimensional structure due to hydrogen bonding

(D) It is a weak electrolyte in water.

Ans. **[D]**

Sol. [B] it does not forms chelate complex with ethylene glycol and become more acidic

[C] it has 2'D' layered structure due to the hydrogen bond.

[D] weak electrolyte in water

29. For the reaction :

$$I^- + ClO_3^- + H_2SO_4 \rightarrow Cl^- + HSO_4^- + I_2$$

The correct statement(s) in the balanced equation is/are

(A) Stoichiometric coefficient of HSO_4^- is 6.

(B) Iodide is oxidized.

(C) Sulphur is reduced.

(D) H_2O is one of the products.

Ans. **[ABD]**

Sol. Balanced reaction will be

$$6I^- + ClO_3^- + 6H_2SO_4 \rightarrow Cl^- + 6HSO_4^- + 3I_2 + 3H_2O$$

$\therefore$ [A], [B] & [D]

30. The pair(s) of reagents that yield paramagnetic species is/are

(A) Na and excess of NH_3

(B) K and excess of O_2

(C) Cu and dilute HNO_3

(D) O_2 and 2-ethylanthraquinol

Ans. **[ABC]**

Sol. [A] $Na + (x+y)NH_3 \longrightarrow [Na(NH_3)_x]^+ + y\ e^-$ (paramagnetic)

Blue colour solution

[B] $K + O_2 \longrightarrow K^{\oplus}O_2^-$ (paramagnetic)

[C] $Cu + \text{dil } HNO_3 \longrightarrow Cu(NO_3)_2 + H_2O + NO$ (paramagnetic)

[D] O_2 + 2-ethylanthraquinol will give H_2O_2 (diamagnetic)

SECTION – II

(One Integer Value Correct Type)

This section contains **10 questions**. Each question, when worked out will result in one integer from 0 to 9 (both inclusive).

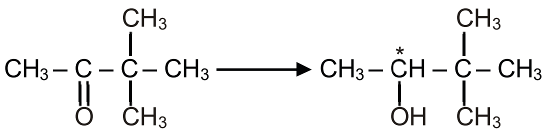
31. Consider all possible isomeric ketones, including stereoisomers of MW = 100. All these isomers are independently reacted with $NaBH_4$ (NOTE : Stereoisomers are also reacted separately). The total number of ketones that give a racemic product(s) is / are

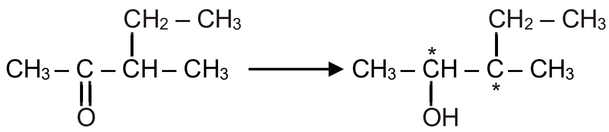
Ans. **[5]**

Sol. Ketone Alcohol

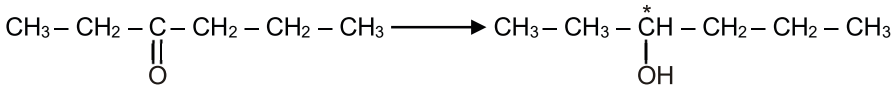
(I) $CH_3-\underset{\underset{O}{||}}{C}-CH_2-CH_2-CH_2-CH_3 \xrightarrow{NaBH_4} CH_3-\underset{\underset{OH}{|}}{\overset{*}{C}H}-CH_2-CH_2-CH_2-CH_3$ Racemic mixture

(II) $CH_3-\underset{\underset{O}{||}}{C}-CH_3-\underset{\underset{CH_3}{|}}{CH}-CH_3 \xrightarrow{NaBH_4} CH_3-\underset{\underset{OH}{|}}{\overset{*}{C}H}-CH_2-\underset{\underset{CH_3}{|}}{CH}-CH_3$ Racemic mixture

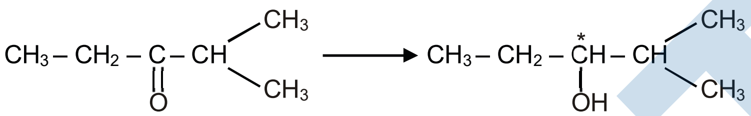
(III)  Racemic mixture

(IV & V) 

+ & – both gives distereomers

(VI) 

Racemic mixture

(VII) 

Racemic mixture

32. A list of species having the formula XZ_4 is given below.

XeF_4, SF_4, SiF_4, BF_4^-, BrF_4^-, $[Cu(NH_3)_4]^{2+}$, $[FeCl_4]^{2-}$, $[CoCl_4]^{2-}$ and $[PtCl_4]^{2-}$

Defining shape on the basis of the location of X and Z atoms, the total number of species having a square planar shape is

Ans. **[4]**

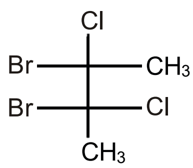
Sol. XeF_4, BrF_4^-, $[Cu(NH_3)_4]^{2+}$, $[PtCl_4]^{2-}$

33. Among PbS, CuS, HgS, MnS, Ag_2S, NiS, CoS, Bi_2S_3 and SnS_2, the total number of BLACK coloured sulphides is

Ans. **[7]**

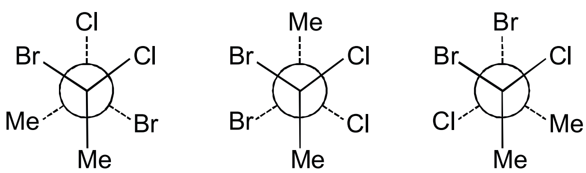
Sol. PbS, CuS, HgS, Ag_2S, NiS, CoS, Bi_2S_3

34. The total number(s) of **stable** conformers with **non-zero** dipole moment for the following compound is (are)



Ans. **[3]**

Sol. Stable conformers $\rightarrow$ Only Gauche form is considered



35. Consider the following list of reagents:

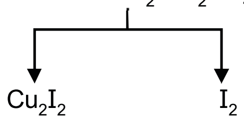
Acidified $K_2Cr_2O_7$, alkaline $KMnO_4$, $CuSO_4$, H_2O_2, Cl_2, O_3, $FeCl_3$, HNO_3 and $Na_2S_2O_3$. The total number of reagents that can oxidise aqueous iodide to iodine is

Ans. **[7]**

Sol. $K_2Cr_2O_7 + H_2SO_4 + KI \longrightarrow K_2SO_4 + Cr_2(SO_4)_3 + H_2O + I_2$

$KMnO_4 + KOH + KI \longrightarrow KIO_3 + K_2MnO_4 + H_2O$

$CuSO_4 + KI \longrightarrow CuI_2 + K_2SO_4$

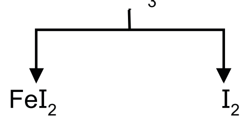


$H_2O_2 + KI \longrightarrow KOH + I_2 + H_2O$

$KI + Cl_2 \longrightarrow KCl + I_2$

$KI + O_3 + H_2O \longrightarrow KOH + I_2$

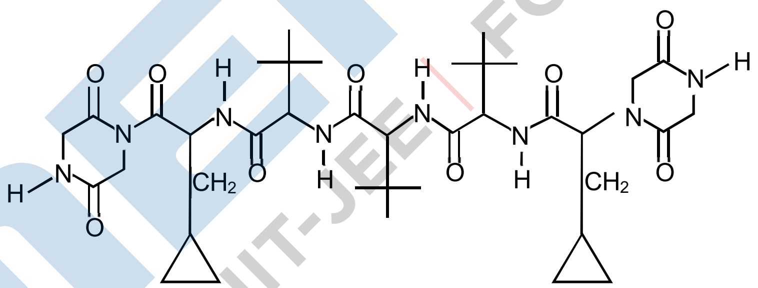
$FeCl_3 + KI \longrightarrow FeI_3 + KCl$



$KI + HNO_3 \longrightarrow KNO_3 + H_2O + I_2 + [NO]$

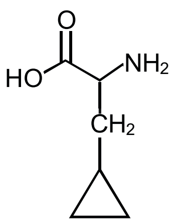
$Na_2S_2O_3$ is acts as a reducing agent.

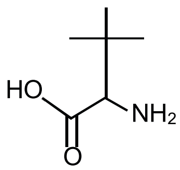
36. The total number of ***distinct naturally occuring amino acids*** obtained by complete acidic hydrolysis of the peptide shown below is

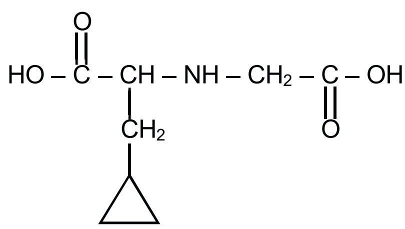


Ans. **[1]**

Sol. (i) $NH_2–CH_2–COOH$

(ii) 

(iii) 

(iv) 

Total 8 molecules of 4 types are obtained on hydrolysis out of which only 1 is naturally occuring amino acid glycine ($NH_2–CH_2–COOH$)

37. In an atom, the total number of electrons having quantum numbers n = 4, $|m_\ell| = 1$ and $m_s = -\frac{1}{2}$ is

Ans. **[6]**

Sol. $n = 4 \quad |m_\ell| = 1 \quad m_s = -\frac{1}{2}$

$\ell = 0 \quad \ell = 1 \quad \ell = 2 \quad \ell = 3$

× 2 2 2 = 6 Ans.

38. If the value of Avogadro number is 6.023×10^{23} mol^{-1} and the value of Boltzmann constant is 1.380×10^{-23} JK^{-1}, then the number of significant digits in the calculated value of the universal gas constant is

Ans. **[4]**

Sol. $K = \frac{R}{N_A}$

$R = KN_A = 6.023 \times 10^{23} \times 1.380 \times 10^{-23}$

Answer 4, on multiplication number of significant digit should be least

39. A compound H_2X with molar weight of 80 g is dissolved in a solvent having density of 0.4 g ml^{-1}. Assuming no change in volume upon dissolution, the molality of a 3.2 molar solution is

Ans. **[8]**

Sol. Assume volume of solution = 1 litre

number of moles of H_2X = 3.2

weight of solvent = 1000 × 0.4 = 400 gm

$m = \frac{3.2}{400/1000} = 8$

40. MX_2 dissociates into M^{2+} and X^- ions in an aqueous solution, with a degree of dissociation (α) of 0.5. The ratio of the observed depression of freezing point of the aqueous solution to the value of the depression of freezing point in the absence of ionic dissociation is

Ans. **[2]**

Sol. $MX_2 \longrightarrow M^{2+} + 2X^-$

1

0.5 0.5 1 i = 2

$\frac{\Delta T_f}{\Delta T_f} = i = 2$

PART C : MATHEMATICS

SECTION – 1

(One or More than One options Correct Type)

This section contains **10 multiple choice questions**. Each question has four choice (A), (B), (C) and (D) out of which **ONE or MORE THAN ONE** are correct.

41. Let M and N be two 3 × 3 matrices such that MN = NM. Further, if $M \neq N^2$ and $M^2 = N^4$, then

(A) determinant of $(M^2 + MN^2)$ is 0.

(B) there is a 3 × 3 non-zero matrix U such that $(M^2 + MN^2)$ U is the zero matrix.

(C) determinant of $(M^2 + MN^2) \geq 1$.

(D) for a 3 × 3 matrix U, if $(M^2 + MN^2)$ U equals the zero matrix then U is the zero matrix.

Ans. **[AB]**

Sol. $M^2 - N^4 = \mathbf{O}$ $\Rightarrow (M - N^2)(M + N^2) = \mathbf{O}$ ......(1)

$\therefore |M - N^2|\,|M + N^2| = 0$

So, $|M + N^2| = 0 \Rightarrow |M|\,|M + N^2| = 0 \Rightarrow |M^2 + MN^2| = 0 \Rightarrow$ Option (A) is correct

(If $|M + N^2| \neq 0$ then $(M + N^2)^{-1}$ exist. So, from eq (1), $M - N^2 = O$, which is contradiction)

Also from Eq.(1), $(M + N^2)(M - N^2) = O$

$\Rightarrow (M^2 + MN^2)(M - N^2) = O \Rightarrow (M^2 + MN^2)\,U = O$, where $U = M - N^2 \Rightarrow$ Option (B) is correct

42. For every pair of continuous functions $f, g : [0, 1] \to R$ such that

$\max\{f(x) : x \in [0,1]\} = \max\{g(x) : x \in [0,1]\}$,

the correct statement(s) is/are

(A) $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0, 1]$.

(B) $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0, 1]$.

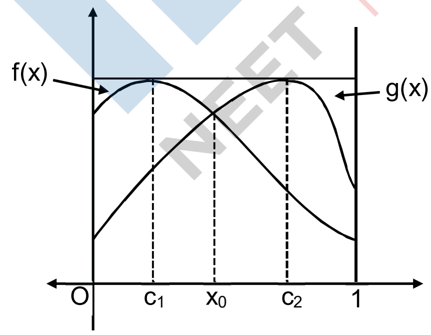
(C) $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$ for some $c \in [0, 1]$.

(D) $(f(c))^2 = (g(c))^2$ for some $c \in [0, 1]$.

Ans. **[AD]**

[one of the possible graphs of (x) & g(x)]

Sol.



$f(x_0) = g(x_0)$

$f(c_1) = g(c_2)$

Aliter: Since f (x) and g (x) are 2 continuous function in [0, 1]

$\therefore$ $\exists\ \alpha, \beta \in [0, 1]$

such that f (α) = M and g (β) = M (where M is the maximum value)

Now consider a function h (x) = f (x) – g (x) in [α, β]

$h(\alpha) = M - g(\alpha) \geq 0$

$h(\beta) = f(\beta) - M \leq 0$

$\therefore$ $h(\alpha) \cdot h(\beta) \leq 0$

$\Rightarrow$ Using I.V.T. $\exists\ c \in [0, 1]$ such that h (c) = 0

hence f (c) = g (c) $\Rightarrow$ (A) & (D).

43. Let f : $(0, \infty) \to R$ be given by $f(x) = \int_{\frac{1}{x}}^{x} e^{-\left(t+\frac{1}{t}\right)} \frac{dt}{t}$.

Then

(A) f(x) is monotonically increasing on $[1, \infty)$.

(B) f(x) is monotonically decreasing on (0, 1).

(C) $f(x) + f\left(\frac{1}{x}\right) = 0$, for all $x \in (0, \infty)$.

(D) $f(2^x)$ is an odd function of x on R.

Sol. **[ACD]**

Replacing $x \to \frac{1}{x}$

$$f\left(\frac{1}{x}\right) = \int_{x}^{\frac{1}{x}} e^{-\left(t+\frac{1}{t}\right)} \frac{dt}{t} = -\int_{\frac{1}{x}}^{x} e^{-\left(t+\frac{1}{t}\right)} \frac{dt}{t}$$

$\therefore f(x) = -f\left(\frac{1}{x}\right)$

$f(x) + f\left(\frac{1}{x}\right) = 0. \Rightarrow$ (C)

Applying Leibnitz rule

$f'(x) = \left(\frac{2}{x}\right) e^{-\left(x+\frac{1}{x}\right)}$

$\therefore f'(x) > 0$

Thus, f(x) is monotonically increasing $\Rightarrow$ (A)

$$f(2^x) = \int_{2^{-x}}^{2^x} e^{-\left(t+\frac{1}{t}\right)} \frac{dt}{t}$$

$$f(2^{-x}) = \int_{2^x}^{2^{-x}} e^{-\left(t+\frac{1}{t}\right)} \frac{dt}{t} = -\int_{2^{-x}}^{2^x} e^{-\left(t+\frac{1}{t}\right)} \frac{dt}{t}$$

$f(2^{-x}) = -f(2^x) \Rightarrow$ (D)

44. Let $a \in R$ and let $f : R \to R$ be given by $f(x) = x^5 - 5x + a$.

Then

(A) f(x) has three real roots if $a > 4$.

(B) f(x) has only one real root if $a > 4$.

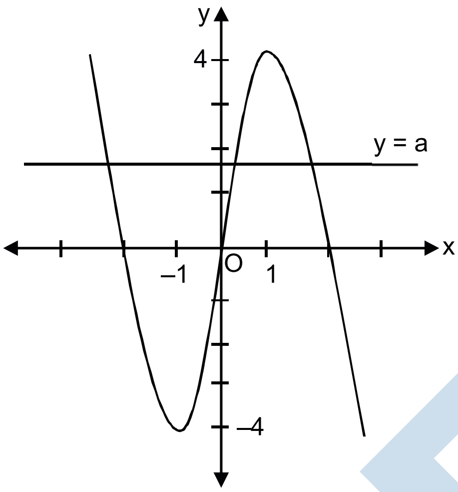
(C) f(x) has three real roots if $a < -4$.

(D) f(x) has three real roots if $-4 < a < 4$.

Ans. **[BD]**

Sol. $a = 5x - x^5 = f(x)$

Now, graph of $f(x) = 5x - x^5$.



From the graph ⇒ (B) & (D)

45. Let $f : [a, b] \to [1, \infty)$ be a continuous function and let $g : R \to R$ be defined as

$$g(x) = \begin{cases} 0, & \text{if } x < a \\ \int_a^x f(t)\,dt, & \text{if } a \le x \le b \\ \int_a^b f(t)\,dt, & \text{if } x > b \end{cases}$$

Then

(A) g(x) is continuous but not differentiable at a.

(B) g(x) is differentiable on R.

(C) g(x) is continuous and not differentiable at b.

(D) g(x) is continuous and differentiable at either a or b but not both.

Ans. **[AC]**

Sol. Checking differentiability at x = a and x = b

$$g'(a^+) = \lim_{h\to 0}\left(\frac{g(a+h)-g(a)}{h}\right) = \lim_{h\to 0}\left(\frac{\int_a^{a+h} f(t)\,dt - 0}{h}\right) = \lim_{h\to 0}\frac{f(a+h)}{1} \Rightarrow g'(a^+) = f(a)$$

$$g'(a^-) = \lim_{h\to 0}\frac{g(a-h)-g(a)}{-h} = \lim_{h\to 0}\frac{0-0}{-h} = 0$$

|||^{ly} $g'(b^+) = 0$ and $g'(b^-) = f(b)$

But $f(a)$ or $f(b) \neq 0$ as co-domain of $f(x) \in [1, \infty)$

$\therefore$ non-derivable at $x = a$ and b.

46. Let $f\left(\frac{-\pi}{2}, \frac{\pi}{2}\right): \to R$ be given by $f(x) = (\log(\sec x + \tan x))^3$.

Then

(A) f(x) is an odd function

(B) f(x) is a one-one function

(C) f(x) is an onto function

(D) f(x) is an even function

Ans. [ABC]

Sol. $f(x) = -f(-x)$

odd function

$f'(x) = 3(\log(\sec x + \tan x))^2 (\sec x \tan x + \sec^2 x)$

$= 3(\log(\sec x + \tan x))^2 (\sec x)[\sec x + \tan x]$

$f'(x) > 0$ in $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

say $t = \sec x + \tan x \left(\frac{1+\sin x}{\cos x}\right) = \frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}} = \frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}$

$\Rightarrow T = \tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$

$x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right) \Rightarrow x \in \left(\frac{-\pi}{4}, \frac{\pi}{4}\right) \Rightarrow \frac{\pi}{4}+\frac{x}{2} \in \left(0, \frac{\pi}{2}\right)$

$\therefore$ $t \in (0, \infty) \Rightarrow y = (\log t)^3 \in (-\infty, \infty)$

$\therefore$ Onto function.

47. From a point P $(\lambda, \lambda, \lambda)$, perpendiculars PQ and PR are drawn respectively on the lines $y = x$, $z = 1$ and $y = -x$, $z = -1$. If P is such that $\angle QPR$ is a right angle, then the possible value(s) of λ is(are)

(A) $\sqrt{2}$ (B) 1 (C) – 1 (D) $-\sqrt{2}$

Sol. C

$L_1: \frac{x}{1} = \frac{y}{1} = \frac{z-1}{0} = \ell$

$L_2: \frac{x}{-1} = \frac{y}{1} = \frac{z+1}{0} = m$

$L_1 \perp L_2$

Dr's of PR = $\lambda + m, \lambda - m, \lambda + 1$

Dr's of PR = $\lambda - \ell, \lambda - \ell, \lambda - 1$

$-1(\lambda + m) + \lambda - m + 0 = 0 \Rightarrow m = 0$

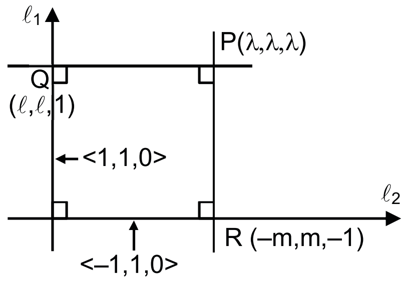
$\lambda - \ell + \lambda - \ell + 0 = 0 \Rightarrow \ell = \lambda$

$PQ \perp PR$

$\therefore (\lambda - \ell)(\lambda + m) + (\lambda - m)(\lambda - \ell) + (\lambda + 1)(\lambda - 1) = 0$

$\Rightarrow \lambda^2 = 1 \Rightarrow \lambda = \pm 1.$

Note: $\lambda = 1$ (rejected) as P = Q = (1, 1, 1).



17. Let $\vec{x}, \vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$.

If $\vec{a}$ is a nonzero vector perpendicular to $\vec{x}$ and $\vec{y} \times \vec{z}$ and $\vec{b}$ is a nonzero vector perpendicular to $\vec{y}$ and $\vec{z} \times \vec{x}$, then

(A) $\vec{b} = (\vec{b} \cdot \vec{z})(\vec{z} - \vec{x})$

(B) $\vec{a} = (\vec{a} \cdot \vec{y})(\vec{y} - \vec{z})$

(C) $\vec{a} \cdot \vec{b} = -(\vec{a} \cdot \vec{y})(\vec{b} \cdot \vec{z})$

(D) $\vec{a} = (\vec{a} \cdot \vec{y})(\vec{z} - \vec{y})$

Sol. ABC

$\vec{a} = \lambda[\vec{x} \times (\vec{y} \times \vec{z})] = \lambda[(\vec{x} \cdot \vec{z})\vec{y} - (\vec{x} \cdot \vec{y})\vec{z}]$

$\vec{a} = \lambda[\vec{y} - \vec{z}]$

take dot product with $\vec{y}$

$\vec{a} \cdot \vec{y} = \lambda[\vec{y} \cdot \vec{z} - \vec{y} \cdot \vec{z}] \Rightarrow \lambda = \vec{a} \cdot \vec{y}$

$\therefore \vec{a} = (\vec{a} \cdot \vec{y})(\vec{y} - \vec{z})$

$|||^{ly}$ $\vec{b} = \mu[\vec{y} \times (\vec{z} \times \vec{x})] = \mu[(\vec{y} \cdot \vec{x})\vec{z} - (\vec{y} \cdot \vec{z})\vec{x}]$

$\vec{b} = \mu(\vec{z} - \vec{x})$

take dot product with $\vec{z}$, we get $\mu = \vec{b} \cdot \vec{z}$

$\therefore \vec{b} = (\vec{b} \cdot \vec{z})(\vec{z} - \vec{x})$

and $\vec{a} \cdot \vec{b} = -(\vec{a} \cdot \vec{y})(\vec{b} \cdot \vec{z})$.

49. A circle S passes through the point (0, 1) and is orthogonal to the circles $(x - 1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then

(A) radius of S is 8

(B) radius of S is 7

(C) centre of S is (– 7, 1)

(D) centre of S is (– 8, 1)

Ans. **[BC]**

Sol. $x^2 + y^2 + 2gx + 2fy + c = 0$ is required circle

$\Rightarrow \quad 2g_1g_2 + 2f_1f_2 = c_1 + c_2$

$0 + 0 = c + (-1) \quad \Rightarrow \quad c = 1$ .....(1)

and $x^2 + y^2 - 2x - 15 = 0$

$2(-1)g + 0 = 1 + (-15)$

$-2g = -14$

$g = 7$ .....(2)

$\therefore$ $x^2 + y^2 + 14x + 2fy + 1 = 0$ satisfies (0, 1)

$0 + 1 + 0 + 2f + 1 = 0 \quad \Rightarrow \quad f = -1$

$\therefore$ Circle is $x^2 + y^2 + 14x - 2y + 1 = 0$

$\therefore$ Centre: (–7, 1)

$r = \sqrt{49 + 1 - 1} = 7$

$\therefore$ option (B), (C)

50. Let M be a 2 × 2 symmetric matrix with integer entries. Then M is invertible if

(A) the first column of M is the transpose of the second row of M.

(B) the second row of M is the transpose of the first column of M.

(C) M is a diagonal matrix with nonzero entries in the main diagonal.

(D) the product of entries in the main diagonal of M is not the square of an integer.

Ans. **[CD]**

Sol. $M = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$

$|M| = ac - b^2 \neq 0 \quad \Rightarrow \quad ac \neq b^2 \quad \Rightarrow$ option (C) (D).

SECTION – 2

(One Integer value correct Type)

This section contains 10 questions. Each question when worked out will result one integer from 0 to 9 (both inclusive).

51. Let a, b, c be positive integers such that $\frac{b}{a}$ is an integer. If a, b, c are in geometric progression and the arithmetic mean of a, b, c is b + 2, then the value of $\frac{a^2 + a - 14}{a + 1}$ is :

Ans. **[4]**

Sol. a, ar, ar^2 where a, r $\in$ N

Now, $\frac{a + b + c}{3} = b + 2 \quad \Rightarrow \quad a(r - 1)^2 = 6 \quad \Rightarrow \quad (r - 1)^2 = \frac{6}{a}$

$\therefore$ a = 6 and r = 2

So, $\frac{a^2 + a - 14}{a + 1} = 4$ **Ans.**

52. Let n $\geq$ 2 be an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of n is

Ans. **[5]**

Sol. $\underbrace{{}^nC_2 - \underbrace{n}_{\text{blue lines}}}_{\text{red lines}} = \underbrace{n}_{\text{blue lines}}$

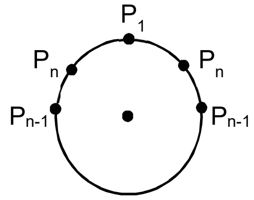
$\Rightarrow$ n = 5. **Ans.**

Aliter: Let there are n points on the circle.

$\Rightarrow$ Number of blue line = n

For red line, choose any point on the circle by ${}^{n}C_1$ ways (says P_1),

now remove p_n and p_2. So number of points left = n – 3



$\therefore$ Number of red line = $\dfrac{{}^{n}C_1 \cdot {}^{n-3}C_1}{2}$

$\therefore$ n = $\dfrac{{}^{n}C_1 \cdot {}^{n-3}C_1}{2}$ = 5. **Ans.**

53. Let $n_1 < n_2 < n_3 < n_4 < n_5$ be positive integers such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$. Then the number of such distinct arrangements $(n_1, n_2, n_3, n_4, n_5)$ is

Ans. **[7]**

Sol. $(n_1 = 1, n_2 = 2, n_3 = 3, n_4 = 5, n_5 = 9)$

$(n_1 = 1, n_2 = 2, n_3 = 3, n_4 = 6, n_5 = 8)$

$(n_1 = 1, n_2 = 2, n_3 = 4, n_4 = 5, n_5 = 8)$

$(n_1 = 1, n_2 = 2, n_3 = 4, n_4 = 6, n_5 = 7)$

$(n_1 = 1, n_2 = 3, n_3 = 4, n_4 = 5, n_5 = 7)$

$(n_1 = 2, n_2 = 3, n_3 = 4, n_4 = 5, n_5 = 6)$

$(n_1 = 1, n_2 = 2, n_3 = 3, n_4 = 4, n_5 = 10)$

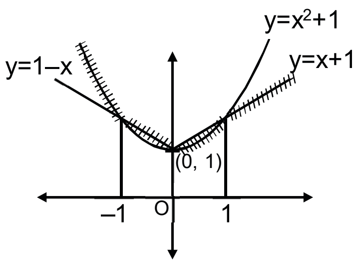
54. Let $f : R \to R$ and $g : R \to R$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1$. Define $h : R \to R$ by

$$h(x) = \begin{cases} \max \{f(x), g(x)\} & \text{if } x \le 0 \\ \max \{f(x), g(x)\} & \text{if } x > 0 \end{cases}.$$

The number of points at which h (x) is not differentiable is

Ans. **[3]**

Sol. Define, $h(x) = \begin{cases} x^2 + 1; & x < -1 \\ -x + 1; & -1 \le x < 0 \\ x^2 + 1; & 0 \le x \le 1 \\ x + 1, & x > 1 \end{cases}$



Clearly, h(x) is non-differentiable at 3 points viz. x = – 1, 0, 1.]

55. The value of $\displaystyle\int_0^1 4x^3 \left\{\frac{d^2}{dx^2}(1-x^2)^5\right\} dx$ is

Ans. **[2]**

Sol. Let $I = \displaystyle\int_0^1 \underbrace{4x^3}_{I} \underbrace{f''(x)}_{II} dx$ where $f(x) = (1 - x^2)^5$ (using I.B.P.)

$= -12 \displaystyle\int_0^1 \underbrace{x^2}_{I} \underbrace{f'(x)}_{II} dx$ (Using I.B.P.)

$$= 24\int_0^1 x\,(1-x^2)^5\,dx$$

Put $\quad 1 - x^2 = t$

$\therefore \quad I = 2$ **Ans.**

56. The slope of the tangent to the curve $(y - x^5)^2 = x(1 + x^2)^2$ at the point $(1, 3)$ is :

Ans. **[8]**

Sol. Differentiate each term on both sides with respect to x, we get

$$2(y - x^2)\left(\frac{dy}{dx} - 5x^4\right) = x \cdot 2(1 + x^2) \cdot 2x + (1 + x^2)^2$$

Put (x = 1, y = 3), we get

$$\left.\frac{dy}{dx}\right]_{(1,3)} = 8 \quad \textbf{Ans.}$$

57. The largest value of the non-negative integer a for which $\lim\limits_{x\to 1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$ is

Ans. **[0]**

Sol. $a \in W$

$$\lim_{x\to 1}\frac{1-x}{1-\sqrt{x}} = \lim_{x\to 1}\left(1+\sqrt{x}\right) = 2$$

$$\therefore \quad \lim_{x\to 1}\left(\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right)^2 = \frac{1}{4}$$

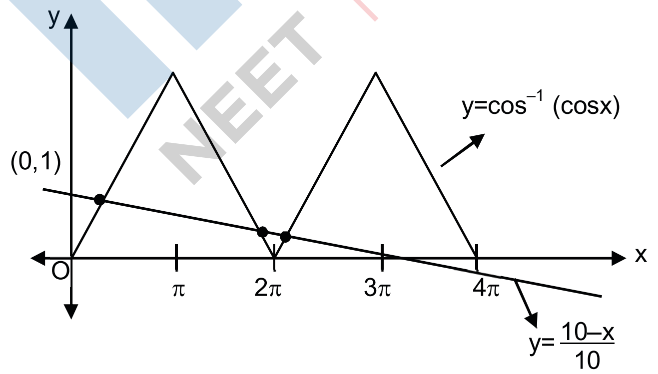
$\Rightarrow \quad \left(\frac{-a+1}{2}\right)^2 = 1 \quad \Rightarrow \quad a = 0, 2$ (a = 2 rejected because base is negative at a = 2)

$\therefore \quad a_{largest} = 0$ **Ans.**

58. Let $f : [0, 4\pi] \to [0, \pi]$ be defined by $f(x) = \cos^{-1}(\cos x)$. The number of points $x \in [0, 4\pi]$ satisfying the equation $f(x) = \frac{10-x}{10}$ is :

Ans. [3]

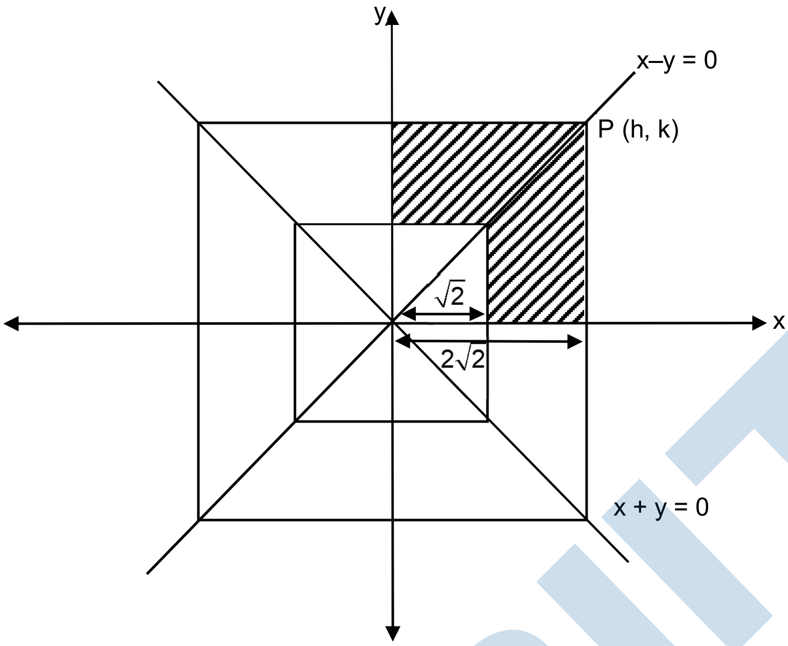
Sol.



Clearly, the equation $\cos^{-1}(\cos x) = \frac{10-x}{10}$ has 3 solutions.

59. For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distances of the point P from the lines $x - y = 0$ and $x + y = 0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \le d_1(P) + d_2(P) \le 4$, is :

Sol.



$\therefore \quad 2 \le \left|\frac{h-k}{\sqrt{2}}\right| + \left|\frac{h+k}{\sqrt{2}}\right| \le 4$

$2\sqrt{2} \le |h-k| + |h+k| \le 4\sqrt{2}$

Required area = $\frac{1}{4}\left(\left(4\sqrt{2}\right)^2 - \left(2\sqrt{2}\right)^2\right) = 6$ (sq. units)

60. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$. If $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$, where p, q and r are scalars, then the value of $\frac{p^2 + 2q^2 + r^2}{q^2}$ is

Ans. 4

Sol. Given $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = \frac{1}{2}$

and $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$ ......(1)

Take dot with $\vec{a}, \vec{b}, \vec{c}$

$\therefore \quad \left[\vec{a}\ \vec{\mathbf{b}}\ \vec{c}\right] = p + \frac{q}{2} + \frac{r}{2}$ .....(2)

$0 = \frac{p}{2} + q + \frac{r}{2}$ .....(3)

$\left[\vec{a}\ \vec{\mathbf{b}}\ \vec{c}\right] = \frac{p}{2} + \frac{q}{2} + r$ .....(4)

(4) – (2)⇒ $\quad r - p = 0$

$\therefore \quad r = p$

Put in equation (3), we get $p + q = 0$

Hence $\frac{p^2 + 2q^2 + r^2}{q^2} = \frac{p^2 + p^2 + 2p^2}{p^2} = 4$

Aliter : $|\vec{a}| = 1 = |\vec{b}| = |\vec{c}|$

Also, $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = \frac{1}{2}$

Now $\vec{a}\times\vec{b} + \vec{b}\times\vec{c} = p\vec{a} + a\vec{b} + r\vec{c}$

Take dot product with $\vec{a}, \vec{b}, \vec{c}$

$\therefore$ on solving, we get

$p = \frac{1}{\sqrt{2}},\ q = \frac{-1}{\sqrt{2}},\ r = \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{p^2 + 2q^2 + r^2}{q^2} = 4$